Selection for Carcass and Feedlot Traits Considering Alternative Slaughter End Points and Optimized Management

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ABSTRACT: Profit was defined as a function of the genotype of animals and variables controlled by management. Alternative parameterizations of management variables were examined to compare the effect of controlling age at slaughter, weight at slaughter, or fat depth at slaughter. The various parameterizations are shown to result in equivalent economic weights for genetic variables, provided management variables are optimized for the current genotype. The implication is that economic weights and selection indexes can be conveniently calculated for age constant end points even though commercial production may involve weight or backfat depth constant slaughter points. An example of selection for profit in the feedlot phase of beef production is presented. Three genotype-management combinations were considered. Economic weights and subsequent selection index weights were shown to depend on both average genotypic means and management (feeding and marketing program) factors.

Key Words: Beef Cattle, Genetic Improvement, Selection Index

Introduction

Equations including returns and costs have been used as a basis for obtaining economic weights for developing beef cattle breeding programs for some time (Dickerson, 1970). However, the end points at which traits have been measured have often differed. A time-constant end point was used by Simm et al. (1986). A finish-constant end point was implicit in the work of Ponzoni and Newman (1989) and Newman et al. (1992) and was used by Koots et al. (1991). Amer et al. (1994b) described changes in economic values with changes in fatness at a constant market weight. The possibility of different end points being used causes difficulties in knowing at which end point economic weights should be obtained and animals compared.

In addition, variables controlled by management, such as time at which animals are slaughtered, can be manipulated to maximize profit. Such changes in management may partially compensate for genetic differences among animals. For instance, larger, later fattening genotypes may be fed a higher energy ration for a longer time than smaller, earlier fattening genotypes so that both are marketed at similar fat depths. The control of variables by management also needs to be considered when determining breeding objectives and subsequent economic weights, as discussed by Amer et al. (1994a).

The objectives of this paper are, firstly, to present a proof of equivalence of economic weights under different end points, considering the interrelationships of genotype and management; and secondly, to provide an example of the development of economic weights for carcass and feedlot traits in beef production and selection indexes for evaluation of beef bulls.

Materials and Methods

Theory. Profit (P) can be considered a function of the genotype of the animals and variables controlled by management decisions (management variables). That is:

\[ P = f(g, m) \]

where \( g \) = a vector of variables determined by the genotype of the animals; and \( m \) = a vector of management variables. The conventional method of determining the economic weight (\( a \)) for each trait is to use the value of a small change in \( g \) about the current mean \( \bar{g} \), while \( m \) is held at the value (\( m_0 \)) that maximizes \( P \) for the current genotypes (Goddard, 1983). That is,
\[ \mathbf{a} = \frac{\partial f}{\partial \mathbf{g}} (\mathbf{g}, \mathbf{m}_0) \]

where
\[ \frac{\partial f}{\partial \mathbf{m}} (\mathbf{g}, \mathbf{m}_0) = 0. \]

The profit function can be parameterized differently by defining new management variables \((\mathbf{n})\), which are functions of \(\mathbf{g}\) and \(\mathbf{m}\) (i.e., \(\mathbf{n} = n(\mathbf{g}, \mathbf{m})\)). For instance, management might control weight at slaughter that is a function of age at slaughter and growth rate, optimized at \(n_0\). Now
\[ P = f_2 (\mathbf{g}, n) = f_2 (\mathbf{g}, n(\mathbf{g}, \mathbf{m})) = f (\mathbf{g}, \mathbf{m}). \]

Economic weights are as follows:
\[ \mathbf{a}_2 = \frac{\partial f_2}{\partial \mathbf{g}} (\mathbf{g}, n_0). \]

But,
\[ \mathbf{a} = \frac{\partial f}{\partial \mathbf{g}} = \left[ \frac{\partial f_2}{\partial \mathbf{g}} + \left( \frac{\partial f_2}{\partial \mathbf{n}} \right) \frac{\partial \mathbf{n}}{\partial \mathbf{g}} \right] \]
\[ = \frac{\partial f_2}{\partial \mathbf{g}} (\mathbf{g}, n_0) \quad \therefore \quad \frac{\partial f_2}{\partial \mathbf{n}} (\mathbf{g}, n_0) = 0 \]
\[ = \mathbf{a}_2. \]

That is, the economic weights are the same regardless of how the management variables are defined, provided they are optimized for the current genotype by the derivative with respect to the management variable being zero. Alternative slaughter end points represent alternative parameterizations of the management variables. For instance, \(\mathbf{m}\) might be age at slaughter and \(\mathbf{n}\) weight at slaughter. Thus, the economic weights are the same whether calculated at a constant age or weight or fat depth, provided management variables have been optimized so that slaughter occurs at an optimum combination of age, weight, and fat depth for the current genotype. The equivalence of economic weights for different end points may not hold if the management variables are optimized at a value where the derivation is not zero but shows a discontinuity (as can happen under quotas, for instance).

To illustrate this conclusion, consider the following simplistic function:
\[ P = w - 2.5f - .5d \]

where
\[ w = \text{weight (kilograms)} = g_w d, \]
\[ f = \text{fat (millimeters)} = g_f d^2 \]
\(g_w\) and \(g_f\) = genetic parameters for weight and fat that are independent of \(d). \]
\[ P = f(g_w, g_f, d) = g_w d - 2.5 g_f d^2 - .5d. \]

To find the optimum value of the management variable \(d, (d_0),\)
\[ \frac{\partial f}{\partial d} = g_w - 5g_f d_0 - .5 = 0 \]
\[ d_0 = \frac{.5 + g_w}{5g_f}. \]

If \(g_w = 1\) and \(g_f = .001,\)
\[ d_0 = 100. \]

Economic weights are as follows:
\[ \frac{\partial f_2}{\partial g_w} = d_0 = 100 \]
\[ \frac{\partial f_2}{\partial g_f} = -2.5 \frac{d_0^2}{5g_f} = -25,000. \]

If the management variable is not age but weight at slaughter \((w),\) then \(d = w/g_w\) and
\[ P_2 = w - 2.5g_f \frac{w^2}{g_w^2} - .5 \frac{w}{g_w} = f_2(g_w, g_f, w). \]

The optimum value of \(w (w_0)\) is obtained by
\[ \frac{\partial f_2}{\partial w} = 1 - 5g_f \frac{w_0}{g_w} - .5 \frac{w}{g_w} \]
\[ w_0 = \frac{g_w^2}{5g_f} = \frac{.5}{g_w} = 100. \]

Economic weights are as follows:
\[ \frac{\partial f_2}{\partial g_w} = \frac{2(2.5)g_f w_0^2}{g_w^3} + \frac{.5w_0}{g_w^2} \]
\[ = 100 \]
\[ \frac{\partial f_2}{\partial g_f} = -2.5w_0^2 = -25,000 \]

as before.

With weight at slaughter held constant, the benefit from increasing growth rate \((g_w)\) comes from lower
fat (economic value given by the term \(2[2.5]g_w^2g_y^3\)) and earlier slaughter age (economic value given by the term \(.5w_0g_w^2\)). However, because weight at slaughter was held constant at the original optimum value, the benefit from increasing \(g_w\), when \(w\) is held constant, is the same as that received from extra weight sold when age at slaughter is held constant.

To define economic weights in this way for a realistic situation, profit must be modeled as a function of both genotype and management variables. The information necessary to do this may not be available. An alternative is to assume that current commercial practice is approximately optimum and to use survey data to define the current values of management variables.

Once a particular parameterization of management variables is chosen, it may be convenient to transform the genetic variables to, for example, \(y = y(g)\). Then \(P_3 = f_3(y, m)\). The economic weights become:

\[
\frac{\partial f_3}{\partial y} = \left(\frac{\partial g}{\partial y}\right) \frac{\partial f}{\partial g}
\]

The total merit of a genotype \((T)\) as a deviation from the mean is unaltered by the transformation because

\[
T = \left(\frac{\partial f_3}{\partial y}\right) (y - \bar{y}) = \left(\frac{\partial f}{\partial g}\right) \left(\frac{\partial g}{\partial y}\right) (y - \bar{y}) = \left(\frac{\partial f}{\partial g}\right) (g - \bar{g})
\]

for small \((y - \bar{y})\), which is the total merit based on the original genetic variables.

In our example, having chosen the parameterization with fixed age \((d_0)\) at slaughter, it is convenient to use:

\[
y_w = d_0g_w \text{ (weight at slaughter), and } \quad y_f = d_0^2g_f \text{ (fat at slaughter)},
\]

so that

\[
P_3 = f_3(y_w, y_f, d_0) = y_w - 2.5y_f - .5d_0.
\]

Economic weights are as follows:

\[
\frac{\partial f_3}{\partial y_w} = 1
\]

and

\[
\frac{\partial f_3}{\partial y_f} = -2.5.
\]

Now the profit function is defined in terms of commonly measured traits, weight and fat, at constant age.

If one had chosen to hold weight at slaughter constant, a different transformation of genetic variables would have been chosen, namely fat and age at constant weight. However, as shown above, the total merit \((T)\) of each animal would be unaffected. Consequently, a selection index \(b\) based on a vector of measurements \(x\) would be the same regardless of the transformation of genetic variables used to define \(T\). Calculation of the index weights \((b)\) would be easiest if the variables \(x\) were defined to the same endpoints as the \(g\) or \(y\) variables in the objective.

Application. The development of economic weights is illustrated with an example for the feedlot phase of beef production. Profit is defined on a per animal basis as revenue minus costs and is converted to profit per kilogram of carcass weight. Relative economic weights are not affected by this change in the base, provided mean profit is zero (Brascamp et al., 1985) or is corrected for changes in scale (Smith et al., 1986). The values used for this example are based on typical prices for 1993 in Canada. The intent is to illustrate the approach used with realistic although not definitive values.

The profit equation is defined per animal as follows:

\[
\text{Profit} = \text{Revenue} - \text{Costs} = C[P1 - f(C) - f(F) + f(R) + f(Q)] - [(I \times P2) + (D \times P3) + P4 + (S \times P5)],
\]

where

\[
C = \text{carcass weight per animal sold};
\]

\[
P1 = \text{price per kilogram carcass weight, assumed to be $3.70 (Cdn)/kg};
\]

\[
f(C) = \text{a function describing a change in price per kilogram of carcass weight approximated as .0000078 (C - 270.8)^2 based on differences for various weights of carcasses};
\]

\[
f(F) = \text{a function describing a change in price per kilogram of carcass weight as a function of backfat depth (as an indication primarily of reduced value of carcasses for suboptimal levels of finish), approximated as .02879 (F - 7.53)^2};
\]

\[
f(R) = \text{a function describing a change in price per kilogram of carcass weight as a function of retail yield percentage (RY%) calculated as } P1(1 - \text{RY})^{-1} \text{ (RY% - RY%) where RY% is 60% and RY% is calculated as 57.34 - .032C + .212L - .681F where C is carcass weight (kilograms), L is longissimus muscle area (square centimeters), and F is backfat depth (millimeters) (Jones et al., 1989)};
\]
f(Q) = a function describing a change in price per kilogram of carcass weight as a function of quality (as measured by marbling, in this example) and assumed to be valued at $.02 per point of marbling score;

I = weight of feed consumed in the feeding period (kilograms of DM);

P2 = price per kilogram of DM of feed, including labor costs of feed handling, assumed to be $.30/kg;

D = days in the feedlot;

P3 = cost per day, assumed to cover investment and ownership costs, feeding equipment costs, and interest costs on facilities, and assumed to be $.39-animal⁻¹ d⁻¹ based on the ratio of animal per day costs relative to beef prices as described in McMorris et al. (1986);

P4 = price per animal, associated with health, trucking, and marketing costs per animal, and assumed to be $120/animal;

S = weight at the start of the feeding period (kilograms);

P5 = price per kilogram of starting weight, assumed to be $2.11/kg (equivalent to $3.70 per kilogram on a carcass basis if dressing percentage is 57%), assuming similar prices per kilogram of live weight at weaning and at slaughter as found by McMorris et al. (1986).

Three situations involving differing combinations of genotype and management programs were used for illustration. The first situation was considered the base (Case 1); the second involved higher average genetic levels for several variables and a higher level of energy in the feeding program (Case 2); the third involved higher average genetic levels and a similar level of energy as the base (Case 3). In each case, management factors including diet and time of marketing were considered to have been optimized. Average levels of production and relationships among variables are shown in Table 1.

In Case 1, weight at start of feeding was taken as .35, and weight at marketing as .85 times mature cow weight, gain per day as .0021 times market weight, longissimus muscle area as .16 times market weight, and intake as 6 times gain during feeding, in line with variables reported by McMorris and Wilton (1986). For Cases 2 and 3, weight at start of feeding was taken as .38 times mature cow weight, assuming a higher milk yield. For Case 2, it was assumed that a higher energy diet was used resulting in a reduced dry matter intake (taken as 80% of 6 kg of DM/kg of gain) but a similar fat depth at marketing compared with Case 1. Price per kilogram DM was increased from $.30 to $.39. For Case 3, it was assumed that the level of energy in the diet was similar to that for Case 1. It was assumed that the optimum management program would result in number of days on feed being increased by 10%, market weights being correspondingly heavier, and level of backfat being reduced relative to Case 2. It was also assumed for this illustration that gain per day was similar in Case 3 and Case 2 in spite of reduced energy in the diet, reflecting a situation in which genotypes were assumed to be similar but not identical.

Profit converted to a kilogram of carcass weight basis was as follows:

\[
\text{Profit/kg} = \text{Revenue} - \text{Costs} = 3.70 - 0.000078(C - 270.8)^2 - 0.02879(F - 7.53)^2 + 0.6175(57.34 - 0.30I + 0.39D + 120 + 2.11S) - 1/C(0.30I + 0.39D + 120 + 2.11S)
\]

Economic weights for the genetic variables carcass weight (C), fat depth (F), longissimus muscle area (L), marbling (M), intake (I), and starting weight (S) are then obtained by finding

\[
\frac{\partial P}{\partial C}, \frac{\partial P}{\partial F}, \frac{\partial P}{\partial L}, \frac{\partial P}{\partial M}, \frac{\partial P}{\partial I}, \frac{\partial P}{\partial S}
\]

respectively, with age at slaughter as the management variable held constant. For the profit equation defined above,

\[
\frac{\partial P}{\partial C} = -0.000078(2)I + 0.000078(270.8)(2)
\]

\[
\frac{\partial P}{\partial F} = -0.02879(7.53)(2)
\]

Three situations involving differing combinations of genotype-management combinations

<table>
<thead>
<tr>
<th>Item</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight at start of feeding, kg</td>
<td>210</td>
<td>266</td>
<td>266</td>
</tr>
<tr>
<td>Gain during feeding, kg</td>
<td>300</td>
<td>329</td>
<td>386</td>
</tr>
<tr>
<td>Gain per day, kg/d</td>
<td>1.08</td>
<td>1.26</td>
<td>1.26</td>
</tr>
<tr>
<td>Days on feed, d</td>
<td>278</td>
<td>261</td>
<td>306</td>
</tr>
<tr>
<td>Market wt, kg</td>
<td>510</td>
<td>595</td>
<td>652</td>
</tr>
<tr>
<td>Carcass wt, kg</td>
<td>291</td>
<td>339</td>
<td>371</td>
</tr>
<tr>
<td>Fat depth at marketing, mm</td>
<td>7</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Longissimus muscle area at marketing, cm²</td>
<td>81.6</td>
<td>95.2</td>
<td>104.3</td>
</tr>
<tr>
<td>Marbling score, pt</td>
<td>8</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Intake of dry matter, kg</td>
<td>1,800</td>
<td>1,579</td>
<td>2,313</td>
</tr>
</tbody>
</table>

*Case 1 represents a base situation; Case 2 a higher genetic level, higher level of energy in the diet, and higher dry matter intake price; and Case 3 level of genotype similar to Case 2 and level of energy similar to Case 1.*
Table 2. Phenotypic coefficients of variation (CV), heritabilities, and genetic correlations for traits in the breeding objectives and gain on feed

<table>
<thead>
<tr>
<th>Trait</th>
<th>CV</th>
<th>C</th>
<th>F</th>
<th>L</th>
<th>M</th>
<th>I</th>
<th>S</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carcass wt (C)(^{b})</td>
<td>.12</td>
<td>.28</td>
<td>.24</td>
<td>.44</td>
<td>.05</td>
<td>.70</td>
<td>.73</td>
<td>.93</td>
</tr>
<tr>
<td>Fat depth (F)</td>
<td>.25</td>
<td>.44</td>
<td>.00</td>
<td>.35</td>
<td>.16</td>
<td>.24</td>
<td>.19</td>
<td></td>
</tr>
<tr>
<td>Longissimus muscle area (L)</td>
<td>.10</td>
<td>.42</td>
<td>-.21</td>
<td>.18</td>
<td>.49</td>
<td>.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marbling (M)</td>
<td>.34</td>
<td>.38</td>
<td>.09</td>
<td>-.09</td>
<td>.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intake of dry matter (I)</td>
<td>.11</td>
<td>.34</td>
<td>.68</td>
<td>.57</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Starting weight (S)(^{d})</td>
<td>.12</td>
<td>.24</td>
<td>.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gain on feed (G)</td>
<td>.14</td>
<td>.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\)Heritabilities on diagonal.
\(^{b}\)Parameters for carcass weight calculated from C = .57 (S + G).
\(^{c}\)Genetic correlations of C with S and G of .76 and .92, respectively, for Case 2 in which proportion of gain to market weight was lower and of starting weight to market weight higher than Cases 1 and 3.
\(^{d}\)Parameters for starting weight taken as those for weaning weight direct.

\[
\frac{\partial P}{\partial L} = .0617(.212),
\frac{\partial P}{\partial M} = .02,
\frac{\partial P}{\partial I} = -.30,
\frac{\partial P}{\partial L} = \frac{.0617}{C},
\]

There is no dependence of economic weights on population means for either muscle area or marbling. In contrast, economic weights for carcass weight and fat depth depend on the mean of the present population, because the profit function is assumed to be nonlinear in C and F.

The use of these economic weights in developing selection indexes is illustrated by considering an index based on traits measured in a bull test program. Measurements available were assumed to be gain on test for 140 d, backfat depth, longissimus muscle area, marbling at end of test, and feed intake during test. The measurement of gain on test and fat depth has been described by Wilton et al. (1989) for the Ontario Bull Evaluation Program. Ultrasonic techniques to obtain estimates of marbling in live animals have been discussed by Brethour (1994).

The three matrices of genetic and phenotypic parameters required to construct an index and calculate expected genetic progress were as follows:

\[
G = \text{genetic variance-covariance matrix of traits in the breeding objective, in this case traits measured on feedlot cattle slaughtered at the ages and mean performance levels given in Table 1;}
\]
\[
P = \text{phenotypic variance-covariance matrix of measurements in the bull test program; and}
\]
\[
C = \text{covariance between traits in the objective and measurements in the bull test program.}
\]

Heritabilities, genetic correlations, and phenotypic coefficients of variation for the traits in the breeding objective, along with those for gain on feed, were based on Koots et al. (1994a,b) and are shown in Table 2. Parameters for carcass weight were taken as a mathematical function of starting weight and gain on feed. These parameters were assumed to apply to all three cases in Table 1. The genetic variance-covariance matrix (G) was calculated from these parameters and the means given in Table 1.

Phenotypic correlations for traits in the bull test are given in Table 3. The phenotypic variance-covariance matrix (P) was obtained from these phenotypic correlations, the coefficients of variation as given in

Table 3. Phenotypic correlation matrix for traits measured in the bull test program

<table>
<thead>
<tr>
<th>Measurement</th>
<th>F</th>
<th>L</th>
<th>M</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain on test (G)</td>
<td>.18</td>
<td>.28</td>
<td>.13</td>
<td>.50</td>
</tr>
<tr>
<td>Fat depth (F)</td>
<td>-.11</td>
<td>.24</td>
<td>.29</td>
<td></td>
</tr>
<tr>
<td>Longissimus muscle area (L)</td>
<td>.06</td>
<td>.14</td>
<td>.24</td>
<td></td>
</tr>
<tr>
<td>Marbling (M)</td>
<td>.24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intake of dry matter (I)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\)From Koots et al. (1994b).
\(^{b}\)Calculated as the product of the correlations of intake with gain and gain with muscle area.
Table 4. Means for measurements taken on bulls on test and their proportion of means in feedlot production (k)

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Case 1a</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain on test, kg</td>
<td>192</td>
<td>235</td>
<td>235</td>
</tr>
<tr>
<td>Fat depth, mm</td>
<td>6.4</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>Longissimus muscle area, cm²</td>
<td>70.4</td>
<td>87.7</td>
<td>87.7</td>
</tr>
<tr>
<td>Marbling, pt</td>
<td>7.3</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>Intake of dry matter, kg</td>
<td>1,152</td>
<td>1,410</td>
<td>1,410</td>
</tr>
</tbody>
</table>

*See Table 1 for description of Cases 1, 2, and 3.

Table 5. Economic weights for genetic variables for feedlot profit

<table>
<thead>
<tr>
<th>Variable</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carcass wt, kg</td>
<td>.0121</td>
<td>.0091</td>
<td>.0074</td>
</tr>
<tr>
<td>Fat depth, mm</td>
<td>-.0115</td>
<td>-.0115</td>
<td>.0461</td>
</tr>
<tr>
<td>Longissimus muscle area, cm²</td>
<td>.0131</td>
<td>.0131</td>
<td>.0131</td>
</tr>
<tr>
<td>Marbling, pt</td>
<td>.0200</td>
<td>.0200</td>
<td>.0200</td>
</tr>
<tr>
<td>Intake of dry matter, kg</td>
<td>-.0010</td>
<td>-.0012</td>
<td>-.0008</td>
</tr>
<tr>
<td>Starting wt, kg</td>
<td>-.0073</td>
<td>-.0062</td>
<td>-.0057</td>
</tr>
</tbody>
</table>

*See Table 1 for description of Cases 1, 2, and 3, noting the higher dry matter intake price in Case 2 than Cases 1 and 3.

Table 2, and the means as given in Table 4. The means were derived as follows. Those for gain on test and fat thickness were taken from de Rose et al. (1988). The mean for longissimus muscle area was obtained as .16 (weaning weight + 28 [daily gain on test] + gain on test), where .16 is the same proportionate value for muscle area as in the feedlot, weaning weight is equivalent to weight at start of feeding in the feedlot, and 28 (daily gain on test) represents gain in the adjustment period. The mean for marbling was assumed to have the same relationship to fat thickness as in the feedlot. The mean for intake was based on a ratio of 6 kg of DM intake/kg of gain, assuming the ratio of DM:gain in the feedlot also applied to bulls on test. Similar diets for bulls on test were assumed for all three cases.

To derive the covariance matrix (C), it was assumed that the genetic correlation between a measurement in the bull test and the same measurement in the feedlot was 1.0 and that both measurements had the same heritability. Carcass weight was assumed to be .57 live weight. The covariance between trait $T_i$ in the bull test and trait $F_j$ in the feedlot is

$$\text{Cov}(T_i, F_j) = k_i \text{Cov}_g(F_i, F_j)$$

where

$$\text{Cov}_g(F_i, F_j) = \text{genetic covariance between trait } i \text{ in the feedlot and trait } j \text{ in the feedlot},$$

$$k_i = \frac{\bar{T}_i}{\bar{X}_f},$$

$$\text{CV}_f(t) = \text{coefficient of variation for trait } i \text{ in the feedlot (test), and}$$

$$\bar{X}_f(t) = \text{mean for trait } i \text{ in the feedlot (test).}$$

It was assumed that $\text{CV}_t = \text{CV}_f$ for all traits in Table 2, so that

$$k = \frac{\bar{T}_i}{\bar{X}_f}$$

with values as shown in Table 4.

Selection index weights ($\mathbf{b}$) were calculated as $P^{-1}\mathbf{a}$, where $\mathbf{a}$ is the vector of economic weights obtained from the partial derivatives described earlier. Standardized weights were obtained by dividing by the corresponding phenotypic standard deviations in bull test.

Results and Discussion

Economic weights describe the value of a small change in each trait from the current mean. The three cases studied show how economic weights are affected by differences in the current mean. There is no intention to compare the profitability of the three cases. It is assumed that all three represent optimum management for a particular combination of genotype (e.g., breed) and prices. In fact, all three have mean profits close to zero.

The economic weights for carcass weight decreased as the average carcass weight increased for Cases 1 to 3 (Table 5). Such a decrease is expected because the carcass weights are increasingly greater than the weight that maximizes price per kilogram. The economic weights are always positive despite carcass weight being above the point of maximum price, because greater carcass weight means reduced costs per kilogram of carcass weight.

In Cases 1 and 2, the economic weight for fat depth was negative but very small. This results from fat depth at slaughter being just below the maximum
price per kilogram \([f(F)]\). In Case 3, the economic weight was positive because slaughter is at 6 mm of fat depth and price per kilogram is maximized at 7.5 mm. In all cases, the economic weight for fat depth was also influenced by the effect of fat depth on retail yield.

Economic weights for muscle area and marbling did not change from case to case because the derivatives involved only retail yield and marbling, respectively. The economic weight for intake changed slightly from case to case, being smaller for Case 3 with greater carcass weight than Case 1 with smaller carcass weight.

The profit equation used was intended to serve as an example. The value of a change in marbling score may differ in various markets. The quadratic function used for carcass weight may also vary with markets, the current example involves a fairly rapid decrease in value per kilogram above approximately 350 kg. Similarly, the quadratic function used for fat depth involves a fairly rapid decrease in value per kilogram below approximately 5 mm.

Amer and Fox (1992) have argued that economic weights should be calculated while \(m\) is varied to be always at the optimum value for each genotype. However, Goddard (1983) pointed out that continually reoptimizing management variables does not alter the economic weight (to a first-order approximation) as shown below.

Let \(m = m(g)\) be a function of genotype that specifies the optimum \(m\) for that genotype. Then \(P = f(g, m(g))\) is profit as a function of \(g\) only with \(m\) optimized. The economic weights are as follows:

\[
\frac{\partial f}{\partial g} = \frac{\partial f}{\partial m} \frac{\partial m}{\partial g} \quad \text{and} \quad \frac{\partial f}{\partial m} = a
\]

as before, because

\[
\frac{\partial f}{\partial m} = 0 \text{ at } (g, m_0).
\]

For instance, this means that the value of a small genetic change in growth rate is the same whether the age at slaughter is held constant at the current optimum value or adjusted to the new optimum.

The economic weights relate to small changes in the mean, as appropriate to genetic change in a population. If large genetic changes are considered, the management should be re-optimized for the new genotype and the full profit function rather than a linear approximation should be used (Goddard, 1983).

The importance of the concept of optimized management is illustrated by the three cases. Case 2 assumed a diet higher in energy (and in price) to suit a larger breed or crossbreed. Case 3 assumed an alternative management system in which diet remained constant, whereas market weight increased and fat depth at marketing decreased. The change in economic weights in Table 5 illustrates the importance of correctly specifying the optimum production program. For instance, if the management of Case 2 was being used, although that of Case 3 would have been more profitable, the economic weight of fat depth would be underestimated. An effect of suboptimal management practices on deriving economic weights was shown in dairy cattle by Dekkers (1991). As discussed earlier, optimum management can be obtained from a profit function that includes management variables if this is available or by assuming that average production practices are approximately economically optimum.

Standardized selection index weights show gain on test to be the most important selection criterion in Case 1, but in Cases 2 and 3 intake was equally or more important (Table 6). Because all animals were considered to have a record for each of the traits measured and all variables had similar heritabilities, differences in index weights largely reflect differences in economic weights. An exception is gain on test, for which no economic weight was directly determined. The selection index weight for backfat depth was of importance only in Case 3, reflecting the fact that only in Case 3 was the economic value of backfat of importance. The increased importance of intake relative to gain on test in Case 3 reflects the decreased economic value of carcass weight but relatively unchanged economic value of intake relative to the other two cases.

The expected total and correlated responses were similar for the three cases, with the exception of fat depth and dry matter intake in Case 3 relative to the other two cases (Table 7). Fat depth would be expected to increase in Case 3. The appropriate change in fat depth depends, of course, on the appropriateness of the pricing situation described in the profit equation. The appropriate change in carcass weight depends on its economic value and relationships with other traits, particularly dry matter intake. In Case 3, the index weights would be expected to lead to an increase in carcass weight and a decrease in dry

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain on test</td>
<td>0.078</td>
<td>0.063</td>
<td>0.060</td>
</tr>
<tr>
<td>Fat depth</td>
<td>0.012</td>
<td>0.010</td>
<td>0.051</td>
</tr>
<tr>
<td>Longissimus muscle area</td>
<td>0.045</td>
<td>0.049</td>
<td>0.057</td>
</tr>
<tr>
<td>Marbling</td>
<td>0.013</td>
<td>0.010</td>
<td>0.013</td>
</tr>
<tr>
<td>Intake of dry matter</td>
<td>-0.062</td>
<td>-0.065</td>
<td>-0.079</td>
</tr>
</tbody>
</table>

\(^a^\)Standardized by division by phenotypic standard deviations in bull test.

\(^b^\)See Table 1 for description of Cases 1, 2, and 3.
matter intake. An improvement in feed efficiency would be expected in all cases, and particularly Case 3, possibly through reduced partitioning of energy to maintenance. Differences from case to case reflect differences both in variances and economic weights, allowing for nonlinearities in pricing and assuming optimized management (including days on feed, diet, and fatness at marketing).

An option of deleting feed intake as one of the measurements available in the bull test was also examined. The expected response in total merit was reduced in all cases (Table 8 compared with Table 7). Major differences in expected changes for carcass weight and intake were found. For example, in Case 3, intake is expected to increase rather than decrease if intake is not included in the traits measured. These results are one example of the possibility of comparing time-constant testing programs on the basis of expected changes in genotypes under optimized management programs. In this example, deletion of feed intake has an important impact. As for all results, this impact depends on the economic situation and genetic parameters assumed.

Economic weights, corresponding selection index weights, and expected correlated responses are calculated on a time-constant basis. As discussed in the derivation of equivalences of economic weights for differing end points, the index weights would be identical to those obtained if economic weights were calculated on a finish or weight constant end point. The derivation presented in this paper thus removes a conceptual problem in considering selection objectives and selection criteria for various end points of expression.

The use of bull test information introduces possible further complexity, in that the length of time on feed is different from that in the feedlot. In addition, bulls are measured instead of steers and the diets used may differ. Our key assumption is that the traits of interest are the same genetically. That is, the genetic correlation between feedlot and bull test is one. For growth rates, evidence of this was provided by Wilton and McWhir (1985). However, more experimental data are required to provide estimates of the genetic covariance between measurements on steers in feedlots and bulls in test stations. These difficulties would be avoided by using progeny tests on steers fed the same diet as feedlot animals would receive, but organizational difficulties would need to be overcome to make such tests practical.

It should be noted that the example considered only profit in the feedlot. To maximize profitability over the whole production system, profit from calf production would have to be added to the profit function used here. This would take into consideration traits such as calving difficulties in terminal lines or maternal abilities in rotational or pure lines. Similarly, discounted gene flow as discussed for various lines by Wilton and Danell (1981) is not incorporated in these examples. It is also important to note that profit functions must be defined at the level of commercial production, usually in terms of crossbred animals.

An alternative to the conventional linear selection index used here is the substitution of estimates of breeding value in the profit equation, as discussed for quadratic profit functions by Wilton et al. (1968). Another option is to incorporate estimates of breeding value into linear programming analyses as suggested by Wilton (1982). Comparisons with this approach are beyond the scope of this paper, but it is interesting to note that linear programming implies concurrent optimization of production under resource constraints and with various genotypes available. The present paper relies on management to have been optimized.

**Implications**

When management variables are optimized, economic weights are equivalent regardless of end point considered. This means that economic weights and selection indexes can be conveniently calculated for age constant end points even though commercial production may use weight or fat depth constant slaughter end points.


**Literature Cited**


